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Natural New Inflation in Broken Supergravity

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Abstract

We consider a natural new inflationary model in broken supergravity based on an R symmetry. The model predicts a concrete relation between the amplitude of primordial density fluctuations and the scale of supersymmetry breaking. The observed value of the density fluctuations is obtained for the gravitino mass of order the weak scale along with a power-law spectral index considerably less than one, which may be tested in future observations.

1 Introduction

Low-energy supersymmetry has attracted much attention in particle physics, since it provides a conceivable solution to the hierarchy problem [1, 2]. Supersymmetric theories naturally accommodate gravity in the form of supergravity [2], which may give us a consistent description of physics below the Planck scale. However, supergravity generically allows a constant term in a superpotential. Thus we expect a negative cosmological constant of order the Planck scale, which yields an anti de Sitter universe. This leads us to seek a further symmetry which avoids such a disastrous situation.

Supersymmetric field theories admit a peculiar symmetry called R symmetry [2]. It is unique in that it can forbid a constant term in a superpotential and thus restrict a cosmological constant in supergravity. It is also ubiquitous in phenomenological models with supersymmetry. Indeed it is a generic ingredient for causing dynamical supersymmetry breaking [3]. These considerations lead us to impose an R symmetry in the framework of supergravity.

In this paper, we consider an R -invariant model of an inflaton where spontaneous breakdown of the R symmetry naturally generates an inflationary universe. The vanishing cosmological constant in the present vacuum implies that the contributions of inflaton potential and supersymmetry breaking sector to the vacuum energy cancel out between each other provided the other contributions are negligible.¹ Then we obtain a concrete relation between the amplitude of primordial density fluctuations and the scale of supersymmetry breaking in our universe. We see that the value of primordial density fluctuations predicted for the gravitino mass of order the weak scale is just around that obtained in the observational analyses. The prediction of the spectral index tends to be considerably less than one for the observed value of the

¹This cancellation requires a fine tuning of parameters, which we postulate in this paper.

density fluctuations, which may be tested in future observations.

2 The model

Let us introduce an inflaton superfield ϕ with R charge $2/(n+1)$, where n denotes a positive integer of order one. Namely, it transforms as

$$\phi(\theta) \rightarrow e^{i\frac{2}{n+1}\alpha} \phi(e^{-i\alpha}\theta). \quad (1)$$

This charge assignment allows a tree-level superpotential

$$W_0 = -\frac{g}{n+1} \phi^{n+1}, \quad (2)$$

where g is a coupling constant of order one. Here and henceforth, we set the gravitational scale $M \simeq 2.4 \times 10^{18}$ GeV equal to unity and regard it as a plausible cutoff in supergravity. Note that the superpotential W_0 by itself yields a fairly flat potential for $n \geq 3$, which is desirable for a slow-roll inflationary scenario. We further assume the presence of a (composite) superfield ² with R charge $2 - 2/(n+1)$ which condenses to give a tiny scale $v^2 \ll 1$. This condensation breaks the $U(1)_R$ symmetry ³ down to a discrete R symmetry Z_{2n} [4]. Then we expect an effective superpotential

$$W = v^2 \phi - \frac{g}{n+1} \phi^{n+1}. \quad (3)$$

The R -invariant effective Kähler potential is given by

$$K = |\phi|^2 + \frac{k}{4} |\phi|^4 + \dots, \quad (4)$$

where k is naturally of order one and assumed to be positive. The ellipsis denotes higher-order terms, which we may ignore in the following analysis.

²This R charge is chosen so that the inflaton ϕ couples to this (composite) superfield.

³This $U(1)_R$ symmetry may be anomalous due to a dynamical origin of the scale v^2 , which avoids the presence of an R axion. We also note that one may impose a discrete R symmetry from the start instead of the continuous one.

The effective potential for the field ϕ in supergravity is given by [2]

$$V = e^K \left\{ \left(\frac{\partial^2 K}{\partial \phi \partial \phi^*} \right)^{-1} |DW|^2 - 3|W|^2 \right\}, \quad (5)$$

where we have defined

$$DW = \frac{\partial W}{\partial \phi} + \frac{\partial K}{\partial \phi} W. \quad (6)$$

This indicates that the vacua ⁴ with $|\phi| < 1$ satisfy the condition [5]

$$DW \simeq v^2 - g\phi^n = 0, \quad (7)$$

which yields a vacuum

$$\langle \phi \rangle \simeq \left(\frac{v^2}{g} \right)^{\frac{1}{n}}. \quad (8)$$

The potential at the vacuum is given by

$$\langle V \rangle = -3e^{\langle K \rangle} |\langle W \rangle|^2 \simeq -3 \left(\frac{n}{n+1} \right)^2 v^4 |\langle \phi \rangle|^2, \quad (9)$$

whose magnitude is much smaller than the inflation scale $V(0) = v^4$.

We propose a scenario that the negative vacuum energy Eq.(9) is canceled out by a supersymmetry-breaking effect which gives a positive contribution Λ^4 to the cosmological constant:

$$-3 \left(\frac{n}{n+1} \right)^2 v^4 \left| \frac{v^2}{g} \right|^{\frac{2}{n}} + \Lambda^4 = 0. \quad (10)$$

This cancellation results in our flat vacuum.⁵ We note that $\Lambda^2 \ll v^2$ for $v^2 \ll 1$. In the hidden sector models of supersymmetry breaking, the scale Λ is chosen so as to give a mass of the weak scale to the gravitino:

$$m_{3/2} \simeq \frac{\Lambda^2}{\sqrt{3}} \simeq 10^{-16} - 10^{-15}. \quad (11)$$

⁴Although these vacua may only correspond to local minima, possible vacua with $|\phi| \geq 1$ do not affect the following analysis.

⁵This is none other than a fine tuning of the cosmological constant, which is the unique unnatural point in the present model. We do not specify the supersymmetry breaking sector since its details are unnecessary for our purposes in this paper (see the final section).

The inflaton mass m_ϕ in the vacuum is given by

$$m_\phi \simeq n|g|^{\frac{1}{n}}v^{2-\frac{2}{n}}. \quad (12)$$

The inflaton ϕ may have the following R -invariant interactions with the ordinary light fields ψ_i in the Kähler potential:

$$K(\phi, \psi_i) = \sum_i \lambda_i |\phi|^2 |\psi_i|^2 + \dots, \quad (13)$$

where λ_i is a coupling constant of order one. The decay width Γ_ϕ of the inflaton is then estimated as

$$\Gamma_\phi \simeq \sum_i \lambda_i^2 |\langle \phi \rangle|^2 m_\phi^3. \quad (14)$$

This decay results in a reheating temperature

$$T_R \simeq g_*^{-\frac{1}{4}} \sqrt{\Gamma_\phi}, \quad (15)$$

where g_* is the relativistic degrees of freedom at the temperature T_R . Hence we get

$$T_R \simeq n^{\frac{3}{2}} |g|^{\frac{2}{n+1}} m_{3/2}^{\frac{3n-1}{2(n+1)}}. \quad (16)$$

3 Inflationary Dynamics

Let us investigate the inflationary dynamics of the above model by means of a slow-roll approximation [6].

We may set $g > 0$ and $\langle \phi \rangle > 0$ without loss of generality and describe the system approximately in terms of the inflaton field φ (≥ 0) which is $\sqrt{2}$ times the real part of the field ϕ . Then the potential for the inflaton reads

$$V(\varphi) \simeq v^4 - \frac{k}{2} v^4 \varphi^2 - \frac{g}{2^{\frac{n}{2}-1}} v^2 \varphi^n + \frac{g^2}{2^n} \varphi^{2n} \quad (17)$$

for $\varphi < \langle \varphi \rangle = \sqrt{2}\langle \phi \rangle$. The k -independent contribution of φ^2 term in $e^K|DW|^2$ is exactly canceled by that in $-3|W|^2$, as was noted in Ref.[4].

The slow-roll inflationary regime is determined by the condition [6]

$$\epsilon(\varphi) = \frac{1}{2} \left(\frac{V'(\varphi)}{V(\varphi)} \right)^2 \leq 1, \quad |\eta(\varphi)| \leq 1, \quad (18)$$

where

$$\eta(\varphi) = \frac{V''(\varphi)}{V(\varphi)}. \quad (19)$$

For the potential Eq.(17), we obtain

$$\begin{aligned} \epsilon(\varphi) &\simeq \frac{1}{2} \left(\frac{-kv^4\varphi - \frac{g}{2^{\frac{n}{2}-1}}nv^2\varphi^{n-1}}{v^4} \right)^2 = \frac{\varphi^2}{2} \left(k + \frac{g^2}{2^{n-1}}nv^{-2}\varphi^{n-2} \right)^2, \\ \eta(\varphi) &\simeq \frac{-kv^4 - \frac{g}{2^{\frac{n}{2}-1}}n(n-1)v^2\varphi^{n-2}}{v^4} = -k - \frac{g}{2^{\frac{n}{2}-1}}n(n-1)v^{-2}\varphi^{n-2}. \end{aligned} \quad (20)$$

The slow-roll condition Eq.(18) is satisfied for $k \leq 1$ and $\varphi \leq \varphi_f$ where

$$\varphi_f \simeq \sqrt{2} \left(\frac{(1-k)v^2}{gn(n-1)} \right)^{\frac{1}{n-2}}, \quad (21)$$

which provides the value of the inflaton field at the end of inflation.⁶ This value is smaller than the vacuum expectation value $\langle \varphi \rangle$ due to $v^2 \ll 1$, which is consistent with the approximation Eq.(17) of the inflaton potential for discussing the inflationary dynamics. The Hubble parameter during the inflation ($0 < \varphi \leq \varphi_f$) is given by

$$H \simeq \sqrt{\frac{V(0)}{3}} \simeq \frac{v^2}{\sqrt{3}}. \quad (22)$$

⁶A sufficiently large expansion of the universe is achieved under the condition that the initial amplitude of the inflaton φ is localized near the origin. We suspect that such an initial condition is derived from some underlying physics. For example, let us consider that the inflation scale v^2 in Eq.(3) arises from a hypercolor quark condensation $\lambda\langle Q\bar{Q} \rangle = v^2$, where the inflaton couples to the hyperquarks as $\lambda Q\bar{Q}\varphi$. If the initial values of Q and \bar{Q} are large as order one, the inflaton field φ is set to be near the origin.

Let us turn to consideration on the horizon of the present universe. The e -fold number N of the present horizon is given by [6]

$$N \simeq 67 + \frac{1}{3} \ln H + \frac{1}{3} \ln T_R \simeq 67 + \frac{1}{3} \ln(n^{\frac{3}{2}} m_{3/2}^{\frac{5n-1}{2(n+1)}}). \quad (23)$$

Let φ_N be the value of the field φ when the observable universe crossed the horizon during the inflation. Then the e -fold number N is also given by

$$N = \int_{\varphi_f}^{\varphi_N} d\varphi \frac{V(\varphi)}{V'(\varphi)}. \quad (24)$$

(i) For $1/n \leq k < 1$, we obtain

$$N \simeq \int_{\varphi_f}^{\varphi_N} d\varphi \frac{v^4}{-kv^4\varphi} = \frac{1}{k} \ln\left(\frac{\varphi_f}{\varphi_N}\right). \quad (25)$$

That is,

$$\varphi_N \simeq \varphi_f e^{-kN}. \quad (26)$$

(ii) For $1/N \leq k < 1/n$, we obtain

$$N \simeq \int_{\bar{\varphi}}^{\varphi_N} d\varphi \frac{v^4}{-kv^4\varphi} + \int_{\varphi_f}^{\bar{\varphi}} d\varphi \frac{v^4}{-\frac{g}{2^{\frac{n}{2}-1}} nv^2 \varphi^{n-1}} = \frac{1}{k} \ln\left(\frac{\bar{\varphi}}{\varphi_N}\right) + \frac{1-nk}{(n-2)k(1-k)}, \quad (27)$$

where $\bar{\varphi}$ is determined by

$$kv^4\bar{\varphi} = \frac{g}{2^{\frac{n}{2}-1}} nv^2 \bar{\varphi}^{n-1}. \quad (28)$$

That is,

$$\varphi_N \simeq \bar{\varphi} e^{-k\bar{N}}, \quad (29)$$

where

$$\bar{\varphi} = \sqrt{2} \left(\frac{kv^2}{gn} \right)^{\frac{1}{n-2}}, \quad \bar{N} = N - \frac{1-nk}{(n-2)k(1-k)}. \quad (30)$$

(iii) We do not consider the region $k < 1/N$ since the coupling k seems unnaturally small for N of several decades.⁷

The value φ_N should exceed the amplitude of quantum fluctuations of the inflaton field in the de Sitter universe [6]:

$$\Delta\varphi \simeq \frac{H}{2\pi} \simeq \frac{v^2}{2\pi\sqrt{3}}. \quad (31)$$

For $n = 3$, we obtain $N \simeq 47$ from Eq.(11) and Eq.(23). The condition

$$\varphi_N \simeq \frac{\sqrt{2}kv^2}{3g} \exp\left(-kN + \frac{1-3k}{1-k}\right) > \Delta\varphi \quad (32)$$

implies that k seems too small to be natural for g of order one. Hence we discard this possibility and restrict ourselves to $n \geq 4$, where the condition $\varphi_N > \Delta\varphi$ is satisfied for a natural range of the parameter k .

4 The Density Fluctuations and Spectral Index

In the above inflationary model, the amplitude of primordial density fluctuations $\delta\rho/\rho$, which arises from quantum fluctuations $\Delta\varphi$ of the inflaton field, is given by [6]

$$\frac{\delta\rho}{\rho} \simeq \frac{3}{5\pi} \frac{H^3}{|V'(\varphi_N)|} \simeq \frac{1}{5\sqrt{3}\pi} \frac{V^{\frac{3}{2}}(\varphi_N)}{|V'(\varphi_N)|} \quad (33)$$

and the spectral index n_s of the density fluctuations is given by [6]

$$\begin{aligned} n_s &\simeq 1 - 6\epsilon(\varphi_N) + 2\eta(\varphi_N) \\ &\simeq 1 - 2k \left\{ 1 + (n-1) \exp\left[-k(n-2)N + \frac{1-nk}{1-k}\right] \right\}. \end{aligned} \quad (34)$$

⁷Roughly speaking, this is the situation analyzed in Ref.[4].

For $1/N \ll k < 1$, we obtain $n_s \simeq 1 - 2k$. The lower bound of the tilt allowed by observations implies $n_s > 0.6$ [7, 8], which is realized for $k < 0.2$. Thus we adopt the range $1/N \leq k < 0.2$ and evaluate the density fluctuations by means of an input Eq.(11).

(a) For $n = 4$, we obtain

$$N \simeq 45, \quad (35)$$

which gives

$$8 \times 10^{-6} g^{\frac{3}{5}} \leq 2 \times 10 g^{\frac{3}{5}} m_{3/2}^{\frac{2}{5}} \leq \frac{\delta\rho}{\rho} < 5 \times 10^3 g^{\frac{3}{5}} m_{3/2}^{\frac{2}{5}} \leq 5 \times 10^{-3} g^{\frac{3}{5}}. \quad (36)$$

The lower and upper bounds correspond to the cases of $k = 1/45$, $m_{3/2} = 10^{-16}$ and $k = 0.2$, $m_{3/2} = 10^{-15}$, respectively.

(b) For $n \geq 5$, we obtain

$$0 < \frac{\delta\rho}{\rho} < 2 \times 10^3 g^{\frac{4}{9}} m_{3/2}^{\frac{5}{9}} \leq 1 \times 10^{-5} g^{\frac{4}{9}}. \quad (37)$$

The observational data yield $\delta\rho/\rho \simeq 2 \times 10^{-5}$ [7, 8], which implies that a realistic inflationary model is given⁸ in the case of $n = 4$. Then the required amplitude of the density fluctuations is obtained for $k \simeq 0.03 - 0.13$ and g of order one, which results in the spectral index $n_s \simeq 0.91 - 0.74$. This tilted power spectrum of the primordial density fluctuations may be adequate for structure formation in our universe [8].

5 Conclusion

Let us summarize the model with $n = 4$. For the gravitino mass Eq.(11) of the weak scale, we obtain the inflation scale $v \simeq 10^{-6}$ and the Hubble

⁸In the case of $n = 5$, the required value $\delta\rho/\rho \simeq 2 \times 10^{-5}$ implies that the spectral index $n_s \simeq 0.6$, which may be marginally consistent with the observations.

parameter during the inflation $H \simeq 10^{-12}$ from Eq.(10) and Eq.(22). The inflaton mass Eq.(12) and the reheating temperature Eq.(16) turn out to be

$$m_\phi \simeq 10^{-9}, \quad T_R \simeq 10^{-16}. \quad (38)$$

For the coupling $k \simeq 0.1$ in the Kähler potential, we get the amplitude $\delta\rho/\rho \simeq 10^{-5}$. The model with the observed amplitude $\delta\rho/\rho \simeq 2 \times 10^{-5}$ predicts a tilted power spectrum of the primordial density fluctuations with the index $n_s \simeq 0.8$.

If the mass of some right-handed neutrino is less than $m_\phi/2$, the inflaton can decay to a pair of the neutrinos through the interaction Eq.(13). In that case, baryogenesis may be followed by leptogenesis from decay of the right-handed neutrino [9] since T_R is of order the weak scale in the above model. The right-handed neutrino with the mass of this order induces, through the seesaw mechanism [10], a tiny mass of a left-handed neutrino in an interesting range for the solution to the solar neutrino problem [11]. We note that the reheating temperature is possibly higher than the weak scale when the inflaton field is involved in a stronger interaction than the one in Eq.(13) [4].

Let us comment on the supersymmetry breaking sector. The Λ^4 contribution to the vacuum energy in Eq.(9) is obtained, for example, by introducing a superfield Z and its superpotential $W(Z) = \Lambda^2 Z$ with an origin of the scale Λ presumably dynamical [12]. During inflation $\varphi \simeq 0$, the field Z acquires a mass of the Hubble scale, which keeps the condition $Z \simeq 0$ at the inflationary epoch⁹ and the contribution of the Z sector is negligible during the inflation for $\Lambda^2 \ll v^2$. Thus the introduction of the field Z scarcely affects the inflationary dynamics. We note that the gravitino mass is possibly as light¹⁰ as 10^{-24} for the lower bound of the spectral index $n_s \simeq 0.6$ though

⁹This may result in a supersymmetry-breaking vacuum without the so-called Polonyi problem [13] if the vacuum lies near the origin $\langle Z \rangle \simeq 0$.

¹⁰Such a light gravitino ($m_{3/2} \simeq 10^{-24}$) is realized in the framework of dynamical

we have regarded $m_{3/2}$ as the weak scale throughout the paper. On the other hand, if we consider the case that the contribution of the inflaton potential to the cosmological constant is canceled by some GUT scale physics instead of the supersymmetry breaking sector, the vacuum expectation value of the inflaton $\langle\varphi\rangle$ turns out to be of order one. Then the present model realizes supersymmetric topological inflation: The model possesses the discrete R symmetry Z_{2n} , which is spontaneously broken to the R parity symmetry by the inflaton condensation $\langle\varphi\rangle \neq 0$. Thus we have n degenerate vacua in this model, which cause domain wall structures in the whole universe. The initial value of the inflaton field $\varphi \simeq 0$ may be naturally achieved in a sufficiently large region inside a domain wall for topological reasons and the resultant defects serve as seeds for inflation [14].

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supersymmetry breaking at low energies. If $n_s > 0.7$ is confirmed in future observations, we will see that our scenario of the vacuum-energy cancelation may be incompatible with the low-energy supersymmetry breaking.

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